Year 12 Mathematics

IAS 2.9

Statistical Inference

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Contents

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NCEA 2 Internal Achievement Standard 2.9 – Statistical Inference

This achievement standard involves using statistical methods to make an inference.

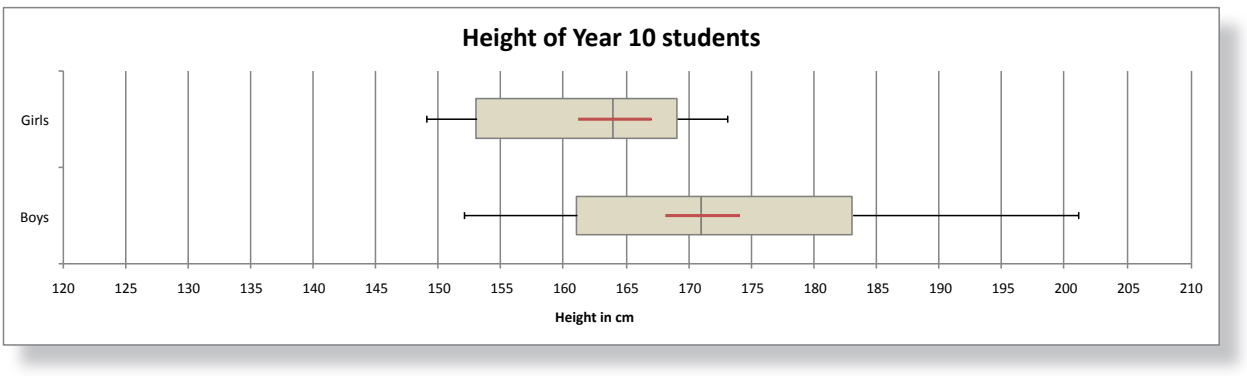
Achievement	Achievement with Merit	Achievement with Excellence
• Use statistical methods to make an inference.	• Use statistical methods to make an inference, with justification.	• Use statistical methods to make an inference, with statistical insight.

- This achievement standard is derived from Level 7 of The New Zealand Curriculum and is related to the achievement objectives
  - carry out investigations of phenomena, using the statistical enquiry cycle:
    - using existing data sets
    - evaluating the choice of sampling and data collection methods used
    - using relevant contextual knowledge, exploratory data analysis, and statistical inference
  - make inferences from surveys:
    - using sample statistics to make point estimates of population parameters
    - recognising the effect of sample size on the variability of an estimate

  in the Statistics strand of the Mathematics and Statistics Learning Area.
- Use statistical methods to make an inference involves showing evidence of using each component of the statistical enquiry cycle to make an inference.

  Use statistical methods to make an inference, with justification, involves linking components of the statistical enquiry cycle to the context, and/or to the population(s), and referring to evidence such as sample statistics, data values, trends, or features of visual displays in support of statements made.

  Use statistical methods to make an inference, with statistical insight, involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle which may involve reflecting on the process, or considering other explanations.
- Using the statistical enquiry cycle to make an inference involves:
  - posing an appropriate comparison or investigative question from a given set of population data
  - selecting random samples
  - selecting and using appropriate displays and measures
  - discussing sample distributions
  - discussing sampling variability, including the variability of estimates
  - making an inference
  - communicating findings in a conclusion.



A Statistical Investigation

The need for Evidence – Is school making New Zealand students shortsighted?

It has long been thought that shortsightedness is mostly a hereditary problem, but researchers led by Professor Ian Morgan of Australian National University say that the environment has a lot more to do with it and that it is getting worse [1].

Reporting in the journal *Lancet*, Professor Morgan and his fellow scientists noted that up to 90% of college students in major East Asian countries, including China, Taiwan, Japan, Singapore and South Korea, are shortsighted. The overall rate of shortsightedness in the United Kingdom, by contrast, is about 20% to 30% and historically the general population in Asia also had similar figures for shortsightedness.

According to the research, the problem is being caused by a combination of factors – a commitment to education and lack of outdoor light.

Professor Morgan argues that many children in South East Asia spend long hours studying at school and doing their homework. This in itself puts pressure on the eyes, but exposure to between two and three hours of daylight acts as a counterbalance and helps maintain healthy eyes.

The scientists believe that a chemical called dopamine could be playing a significant part. Exposure to light increases the levels of dopamine in the eye and this seems to prevent elongation of the eyeball.

"We're talking about the need for two to three hours a day of outdoor light – it doesn't have to be massively sunny, we think the operating range is 10-20,000 lux, we're not sure about that - but that's perfectly achievable on a cloudy day."

Cultural factors also seem to play a part. Across many parts of South East Asia, young school children often have a lunchtime nap. According to Professor Morgan they are missing out on prime light to prevent shortsightedness.

[1] *Professor Morgan's study was widely reported including in*

BBC - "Massive rise in Asian eye damage" 4th May 2012.

http://www.bbc.co.uk/news/health-17942181

Shortsightedness is called Myopia but it is also called nearsightedness.

When a person with this condition looks at a distant object it is out of focus but when they look at a close object it is in focus. People with this condition need to wear glasses or contact lens to clearly see objects that are further away.

The Statistical Problem

The Purpose of the Investigation

What do you hope to demonstrate with your investigation? Are you hoping to make predictions about the population or are you hoping to show that it is likely that one part of the population is different from another part of the population on some measure?

Your investigation should start with a purpose statement and follow this with the investigative comparative question.

A comparative question is one that attempts to see if one group is different in a nominated way from another group.

The purpose statement should contain why you have decided to conduct this investigation, your prediction for the outcome and a definition of the population that you think the prediction applies to.

It could start with the phrase "I would like to know ... and continue with statements similar to "I expect ..." or "I think ...

For Example:

I know that boys in Year 13 are generally bigger than girls so if there is an age when girls are taller than boys then they must have had a growth spurt before boys. I expect as young teenagers (Year 9) that girls will have reached puberty sooner and may be taller. I selected Year 9 as most students in Year 9 are between 12 and 14 so are young teenagers. I do not have access to all the height data of the New Zealand population but I will use the height data for Year 9 students on the CensusAtSchool 2013 data base. As a very large number of New Zealand students have entered height data on the website I believe I have access to data that reflects the population I intend comparing.

The Statistical Question

The question will be one sentence and describe exactly what you will compare. It should start with "I wonder if ..." and should indicate the population and what you will compare. It should indicate the parameter that will be inspected.

For example:

"I wonder if girls in Year 9 tend to be taller than boys in Year 9 in New Zealand? I will look at the median heights of Year 9 girls and Year 9 boys on a sample of heights from the CensusAtSchool 2013 data base."

The question should be precise and state the variable you will measure and the groups you will compare.

Use a Mnemonic to remember the six points that you must define in your Purpose Statement and Comparative Question. You may choose to make your own mnemonic but one possible answer is

C – Comparative Question

P – define your Population

G – the Groups you plan to compare

V – the numeric Variable you will use

M – the parameter such as Median you will use

D – the Direction you expect the difference to be.

Check you have the SIX points. The combination of your Purpose Statement and Comparative Question should include:

1. **C** – Write a Comparison question.
   "I will compare the median heights of Year 9 girls and Year 9 boys ..."
2. **P** – Define the Population from which the sample is taken.
   "... Year 9 students on the 2013 CensusAtSchool data base."
3. **G** – Identify the groups that are being compared.
   "... Year 9 girls and Year 9 boys"
4. **V** – State the numerical Variable that will be used for the comparison.
   "... heights ..."
5. **M** – State which population parameter will be used to see if there is a difference.
   "... the median heights ...."
6. **D** – Indicate what direction the researcher expects the difference (if any) to be.
   "I wonder if girls in Year 9 tend to be taller than boys in Year 9 in New Zealand?

Statistical Ethics

An investigation often requires people to cooperate with you. You should explain what you are investigating, that the data about individuals is anonymous and what you hope to demonstrate.

It should not be used to make one group feel inferior to another group.

Attempts to show negative differences, for example between races or gender, could result in the goodwill that enabled data to be collected in the first place be withdrawn.

If it is possible that people are likely to be offended or upset by your investigation then you should not be conducting it.

Achievement – For each of these statements (which may or may not be true), rewrite them as an investigative question with plausible populations that could be investigated with statistics. Your question should indicate what you will compare to get an answer to your question. Your comparative question could start with "I wonder if ..." but must indicate the groups you will compare, a population, the numeric variable, the population parameter and the direction you believe will be shown to be different.

13. Left-handed students and right-handed students have different abilities in art.

14. Caffeine based drinks affect students' ability to concentrate.

15. Students in bigger cities have different fitness levels than students in small towns.

16. What does the number of text messages a person sends depend upon?

17. Teenagers have a different body clock to adults.

18. What a teenager has for breakfast may affect them.

19. Students at single sex schools achieve better academically.

20. What differences are there between soccer players and rugby players?

Sample Size

Sample Size

On page 26 and 27 we have height data from a population of 1188 Year 11 students (male and female). We know the parameter from our population and we want to explore how well different sample sizes reflect the population statistics.

Use your calculator to generate random numbers from 1 to 1188. You will need different sized samples. As you generate each location using your random numbers mark it with one colour. Stop when you have the required sample size and enter the random data into your calculator to find the minimum, lower quartile, median, upper quartile and maximum.

The population parameters are shown here. These are shown as a box and whisker plot on the next page. Using the same scales plot the statistics on each sample directly underneath. We call these measures parameters for the population and the same measures from a sample are called statistics.

Statistics	Population data
Minimum	123 cm
LQ	161 cm
Median	167 cm
UQ	174 cm
Maximum	201 cm

Merit – You need to take samples at the specified sample size from the population on pages 26 and 27. Calculate the statistics required and plot a box and whisker plot above the graph of the population on the next page. Comment on the sampling variability between samples from the one population.

31. You are interested in taking **three** different samples of size 30 from the 1188 students in the population. For each sample calculate the statistics required and draw a box and whisker plots on the grid on page 25. Comment on similarities and differences between each of your samples and the population box and whisker plot already displayed. Use these results to comment on the sampling variability with a sample size of 30.

	Pop. data	Sample 1 n = 30	Sample 2 n = 30	Sample 3 n = 30
Min.	123 cm			
LQ	161 cm			
Median	167 cm			
UQ	174 cm			
Max.	201 cm			

32. You are interested in taking **two** different samples of size 100 from the 1188 students in the population. For each sample calculate the statistics required and draw a box and whisker plots on the grid on page 25. Comment on similarities and differences between each of your samples and the population box and whisker plot already displayed. How good are the samples of 100 to enable you to describe the population compared to your results and displays from Q31?

	Pop. data	Sample 1 n = 100	Sample 2 n = 100
Min.	123 cm		
LQ	161 cm		
Median	167 cm		
UQ	174 cm		
Max.	201 cm		

Data Sets – SURF for Schools

Statistics New Zealand

SURF for Schools is a Synthetic Unit-Record File containing information generated by either the Household savings survey, the Income survey or the Census. These SURF records are not about real people, but are generated using statistical techniques to have similar characteristics as to those of the respondents to the survey. This modification helps Statistics New Zealand prevent any unintentional disclosure of data about an individual's personal information. The data in the SURF data sets shows many of the same patterns as the original data sets, and analysis will give results that are close to the results from the original survey.

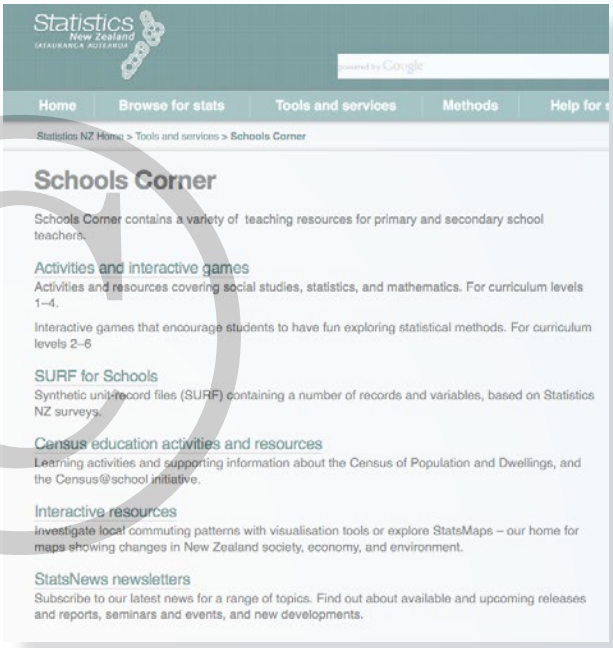
Therefore if the statistical investigation we want to undertake concerns families, savings, incomes and households then this is an excellent source. It also appears 'clean' in that there does not appear to be erroneous data amongst it.

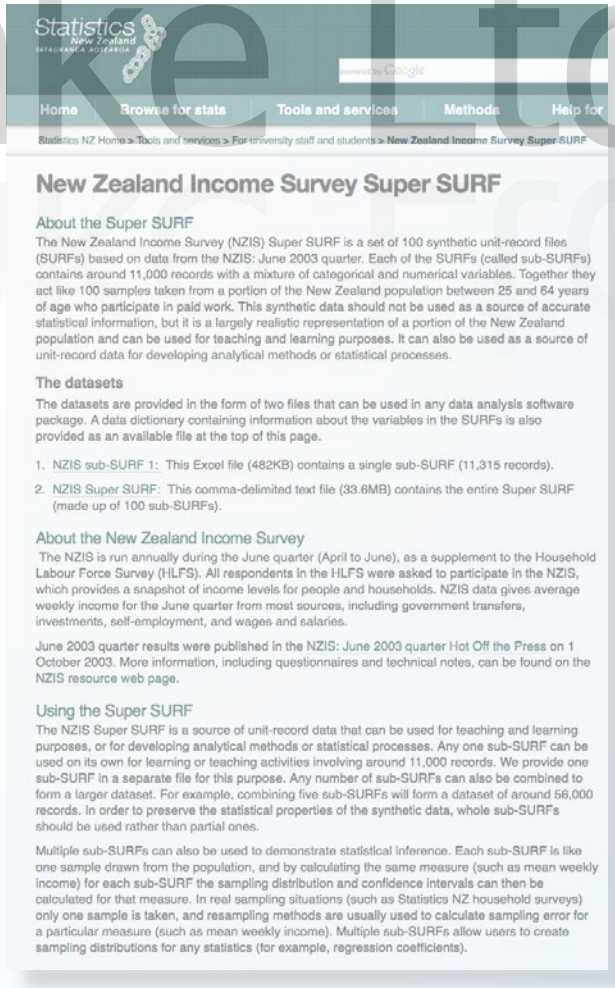
The variables in each data set change and the surveys cover different periods.

- The Household Savings Survey was a nationwide survey that collected information on the net worth (assets minus liabilities) of New Zealanders. As well as data on gender, employment status, qualification level and ethnicity there is information on household income, individual salary, household debt and net worth. (sample of 300 records)
- The New Zealand Income Survey has records on age, gender, qualification level and ethnicity, hours a week worked and weekly income. (sample of 200 records)
- The Census SURF includes information similar to the income survey but also information on cigarette smoking, mobile phones and internet use. (16 regional samples of 300 records).
- The Super SURF is a set of 100 synthetic unit-record files (SURFs) based on data from 2003. Each of the SURFs (called sub-SURFs) contains around 11,000 records with a mixture of categorical and numerical variables.

  Person ID: a random number for each record
  Surf ID: a numerical identifier per sub-SURF
  Age: 25 to 64 years in 5-year age bands
  Sex: male and female
  Ethnicity: six categories
  Highest qualification: five categories
  Weekly hours: total usual weekly hours worked
  Weekly income: total usual weekly income.

Acknowledgment: References to SURF and Statistics New Zealand, screen shots and graphics reprinted with kind permission of Statistics New Zealand. http://www.stats.govt.nz/tools_and_services/university-students/NZIS-Super-SURF.aspx

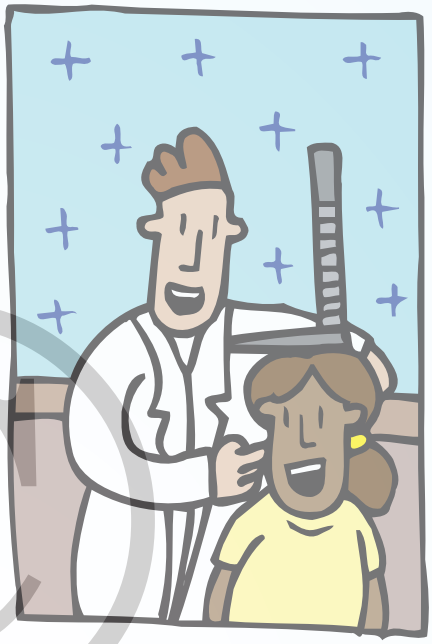




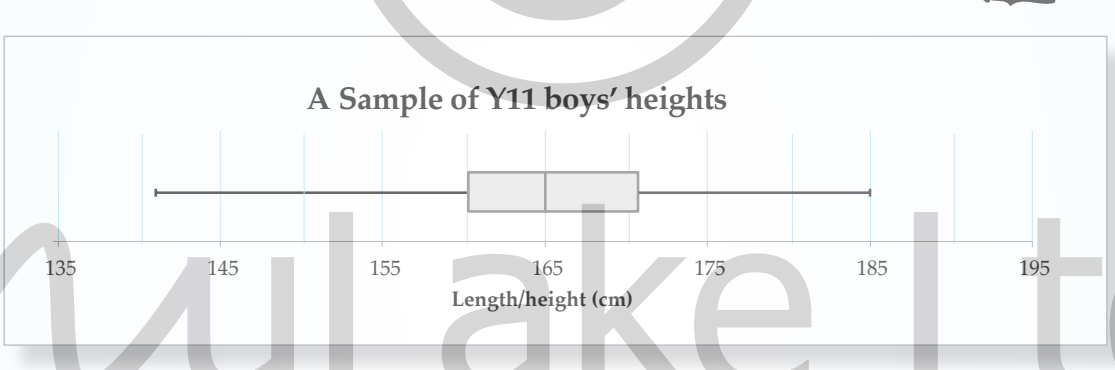
Example

A random sample of Year 11 boys' heights produced the following sample statistics.

Sample size	58
Mean	165.7 cm
Std. Dev.	9.2 cm
Minimum	141.0 cm
Lower Quartile	160.3 cm
Median	165.0 cm
Upper Quartile	170.8 cm
Maximum	185.0 cm



Graph the results on a box and whisker plot and describe the distribution. Calculate the range that the population median is likely to be in and show this range on your box and whisker plot.



The results for this sample of the heights of 58 boys show we have a median height of 165.0 cm. Half of the boys have a height in the range 160.3 cm (lower quartile) to 170.8 cm (upper quartile). The tallest 25% of boys are between 170.8 cm and 185 cm tall. The shortest 25% of boys are between 141.0 cm and 160.3 cm. The distribution is fairly symmetrical with the two quartiles about 5 cm from the median.

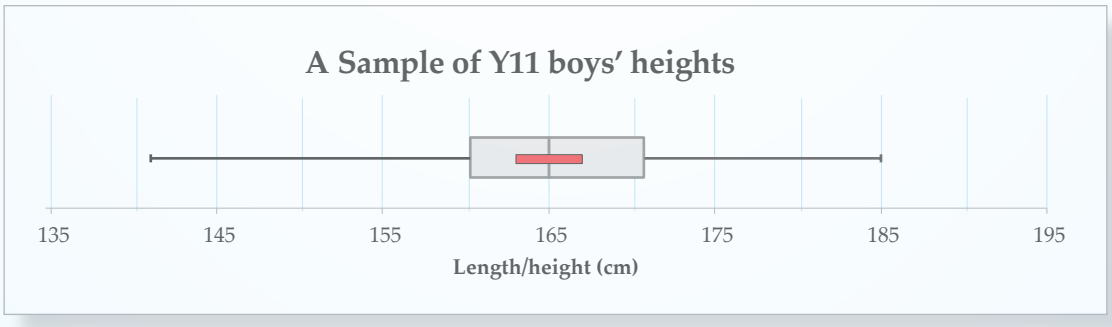
$$\text{Informal CI.} = \text{Median} \pm 1.5\left(\frac{\text{IQR}}{\sqrt{n}}\right)$$

$$\text{Informal CI.} = \text{Median} \pm 1.5\left(\frac{\text{UQ} - \text{LQ}}{\sqrt{\text{sample size}}}\right)$$

$$= 165.0 \pm 1.5\left(\frac{170.8 - 160.3}{\sqrt{58}}\right)$$

$$= 165.0 \pm 2.1 \text{ cm}$$

The interval bar is drawn 2.1 cm either side of the median on the box and whisker plot.
We expect the population median is in the range 162.9 cm to 167.1 cm.



43. Who studies more?
The Principal was given the following statistics of weekly out-of-school homework and study hours from a random sample of Year 11 and Year 13 students Draw a box and whisker plot of these statistics showing the variability (informal confidence interval) in relation to the median. Comment on the graphs and statistics and draw a conclusion as to whether Year 13 students are likely to study more than Year 11 students at this school.
All figures are in hours per week.

	Y11 n = 75	Y13 n = 53
Mean	10.9	13.2
Standard Deviation	9.2	8.7
Minimum	0	0.5
Lower Quartile	2.5	4.5
Median	11.5	18.5
Upper Quartile	18.0	22.5
Maximum	25.0	29.0

44. Who is taller in Year 10?
Draw a box and whisker plot of these statistics showing the variability (informal confidence interval) in relation to the median. Comment on the graphs and statistics and draw a conclusion as to whether males are taller than females in Year 10.

	Boys n = 117	Girls n = 127
Mean	164.9 cm	161.7 cm
Standard Deviation	10.0 cm	8.9 cm
Minimum	137 cm	139 cm
Lower Quartile	156 cm	152 cm
Median	166 cm	161 cm
Upper Quartile	171 cm	165 cm
Maximum	190 cm	183 cm

Page 7

1. Mean = 466.2 (1 dp)
Median = 367
Lower quartile = 276
Upper quartile = 649
IQ range = 373
Range = 840

2. Mean = 53.6 (1 dp)
Median = 52
Lower quartile = 32
Upper quartile = 70
IQ range = 38
Range = 69

3. Mean = 6.43 (2 dp)
Median = 6.365
Lower quartile = 4.27
Upper quartile = 8.82
IQ range = 4.55
Range = 11.05

4. Mean = 24.7 (1 dp)
Median = 21
Lower quartile = 11
Upper quartile = 37
IQ range = 26
Range = 97

5. a) Mean = 5.26 (2 dp)
Median = 5.1
Lower quartile = 3.2
Upper quartile = 7.0
Range = 7.3

b) Mean = 526 (3 sf)
Median = 510
Lower quartile = 320
Upper quartile = 700
Range = 730
All 100 times as each piece of data is multiplied by 100.

6. a) Mean = 54.5 (1 dp)
Median = 53
Lower quartile = 25
Upper quartile = 77
Range = 86

b) Mean = 354.5 (1 dp)
Median = 353
Lower quartile = 325
Upper quartile = 377
Range = 86
Measures of the middle have 300 added but measues of spread are unchanged.

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7. Std. Dev. = 6.66 (2 dp)
Variance = 44.41 (2 dp)

8. Std. Dev. = 5.46 (2 dp)
Variance = 29.84 (2 dp)

9. Mean = 58.75 (2 dp)
Std. Dev. = 16.80 (2 dp)
proportion within one
Std. Dev. is 14 out of 20 = 0.7

Page 11 cont...

10. Mean = 28.45 (2 dp)
Std. Dev. = 6.81 (2 dp)
Proportion within one
Std. Dev. is 12 out of 20 = 0.6

11. a) Mean = 164.60 (2 dp)
Std. Dev. = 8.80 (2 dp)

b) Proportion within one
Std. Dev. is 22 out of 30 = 0.73

12. a) Each measure is doubled so
68, 68, 90, 90, 90, 92, 112, 112, 112, 114, 114, 114, 134, 134, 146.

b) Mean only plus 35 so
69, 69, 80, 80, 80, 81, 91, 91, 91, 92, 92, 92, 102, 102, 108.

c) Mean only subtract 53 to 0 so
$^{-}19$, $^{-}19$, $^{-}8$, $^{-}8$, $^{-}8$, $^{-}7$, 3, 3, 3, 4, 4, 4, 14, 14, 20.

Page 15

13. I wonder if left-handed students in Year 11 at my school gain less achievement credits than right-handed students in art? I will compare the median total art achievement NCEA 1 credits of this year's Year 12 students at my school to the handedness of students.

14. I wonder if Year 11 students who drink caffeine complete more homework than students who do not? I will compare the median number of hours homework of Year 11 students at our school for students who drink less than five caffeine drinks a week compared to students who drink more than five caffeine drinks a week.

15. I wonder if there is less participation in sport per week from students who live in large cities compared to students who live in small towns in New Zealand? I will compare the median total hours participation in sport of Year 11 students on the CensusAtSchool data base for those living in large cities (over 300 000 population) to students in small cities (under 100 000 population).

16. I wonder if the number of texts sent per week is greater for girls than boys? I will compare the median number of text message sent per day of Year 11 girls and boys at our school.

Page 15 cont...

17. I wonder if the median hour of going to bed is greater for teenagers than adults? I will compare the median bedtime on week nights of teenagers at our school to their parents.

18. I wonder if students who usually eat a high starch breakfast gain more credits in NCEA compared to students who have a small breakfast? I will compare the median total achievement NCEA 1 credits of this year's Year 12 students at our school for students who regularly eat muesli, porridge or weet-bix to students who eat something else or nothing.

19. I wonder if students at single sex schools get more achievement standards in Year 11 than students at coeducational schools? I will compare the median total achievement NCEA 1 credits of this year's Year 12 students at the boys and girls schools in my town to students at the two coeducational schools in my town.

20. I wonder if soccer players are lighter than rugby players? I will compare the median weight of boys at my school that play soccer to boys that play rugby.

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Other correct answers are possible.

21. This may be biased as
- from 3pm to 5pm very few people with full-time jobs will be near Countdown.
- you may approach some people who do not live in Napier.
- you may only approach friendly looking people.

22. This may be biased as
- a significant section of the population either does not have a phone or has an unlisted number.
- Wellington people are unlikely to be representative of New Zealand as a whole.
- you have no control on who answers the questions. They may not be of voting age. A lot of teenagers will answer the phone.